

## Comprehensive Mathematics Instruction (CMI) Framework

### DEVELOP UNDERSTANDING

The goal of Develop Understanding is to surface student thinking, which leads to understanding of ideas, strategies, and representations relative to a selected mathematical purpose.

LAUNCH	EXPLORE	DISCUSS
<p><b>PURPOSE:</b> Pose a task with:</p> <ul style="list-style-type: none"> <li>● Clear mathematical purpose aligned with a state or national standard or objective.</li> <li>● Multiple paths to solutions, and/or multiple solutions.</li> </ul> <p><b>TEACHER ROLE:</b> <b>Before</b></p> <ol style="list-style-type: none"> <li>1. Identify mathematical objective(s) for the lesson.</li> <li>2. Select or design an appropriate task, e.g. new tasks, previously posed tasks, student-generated ideas, misconceptions, or questions.</li> </ol> <p><b>During</b></p> <ol style="list-style-type: none"> <li>3. Activate student background knowledge.</li> <li>4. Launch and clarify the task.</li> </ol> <p><b>STUDENT ROLE:</b></p> <ol style="list-style-type: none"> <li>1. Actively listen.</li> <li>2. Ask clarifying questions.</li> <li>3. Access background knowledge.</li> </ol>	<p><b>PURPOSE:</b> Allow students to build understanding of the selected mathematical purpose by engaging in the task through:</p> <ol style="list-style-type: none"> <li>1. Developing ideas.</li> <li>2. Developing problem solving strategies.</li> <li>3. Developing multiple representations using: <ul style="list-style-type: none"> <li>● Appropriate manipulatives and/or technology</li> <li>● Charts, tables, diagrams, pictures, etc.</li> </ul> </li> </ol> <p><b>TEACHER ROLE:</b> <b>Before</b></p> <ol style="list-style-type: none"> <li>1. Anticipate student thinking.</li> <li>2. Determine student grouping (individuals, pairs, small groups).</li> </ol> <p><b>During</b></p> <ol style="list-style-type: none"> <li>3. Allow student exploration and discourse.</li> <li>4. Facilitate exploration by asking questions to: <ul style="list-style-type: none"> <li>● engage students in the task,</li> <li>● prompt or guide student exploration,</li> <li>● clarify mathematical thinking,</li> <li>● deepen student thinking.</li> </ul> </li> <li>5. Assess and select 3 to 5 ideas, strategies, and/or representations to share during Discuss phase. <ul style="list-style-type: none"> <li>● Order by level of complexity to develop connections between ideas, strategies, and/or representations.</li> <li>● May choose incorrect examples to illustrate common misconceptions</li> </ul> </li> </ol> <p><b>STUDENT ROLE:</b></p> <ol style="list-style-type: none"> <li>1. Engage in task.</li> <li>2. Reflect on individual or group work by questioning, explaining, and justifying thinking.</li> <li>3. Ask “Does this make sense?”, “Have I seen something like this before?”</li> <li>4. Seek proper vocabulary in order to explain what is being observed.</li> </ol>	<p><b>PURPOSE:</b> Develop student understanding of ideas, strategies, and representations by having students communicate, explain, and support their own thinking and interact with the thinking of their peers.</p> <p><b>TEACHER ROLE:</b> <b>Before</b></p> <ol style="list-style-type: none"> <li>1. Anticipate the structure and flow of the discussion of selected ideas, strategies, and/or representations.</li> </ol> <p><b>During</b></p> <ol style="list-style-type: none"> <li>2. Orchestrate discussion of selected ideas, strategies, and/or representations.</li> <li>3. Help students understand criteria for judging ideas, strategies, and/or representations.</li> <li>4. Assess while helping students clarify mathematical reasoning behind ideas, strategies, and/or representations.</li> <li>5. Assess while helping students compare and connect ideas, strategies, and representations, using appropriate mathematical vocabulary.</li> <li>6. Help students summarize and connect discussion to the selected mathematical purpose.</li> </ol> <p><b>After</b></p> <ol style="list-style-type: none"> <li>7. Determine the next phase: <ul style="list-style-type: none"> <li>● Remain within Develop Understanding phase, or</li> <li>● Move to Solidify Understanding phase</li> </ul> </li> </ol> <p><b>STUDENT ROLE:</b></p> <ol style="list-style-type: none"> <li>1. Share and explain thinking.</li> <li>2. Actively participate by listening, describing, complimenting, or comparing student work.</li> <li>3. Question to clarify understanding.</li> </ol>

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### SOLIDIFY UNDERSTANDING

The goal of Solidify Understanding is to examine and extend student ideas, strategies, and representations, which leads to a development of concepts, algorithms, and tools.

LAUNCH	EXPLORE	DISCUSS
<p><b>PURPOSE:</b></p> <ol style="list-style-type: none"> <li>Pose a task with a focused <ul style="list-style-type: none"> <li>idea,</li> <li>strategy, and /or</li> <li>representation</li> </ul> </li> <li>Designed to <ul style="list-style-type: none"> <li>confirm,</li> <li>connect,</li> <li>generalize, and/or</li> <li>transfer</li> </ul> mathematical understanding</li> </ol> <p><b>TEACHER ROLE:</b></p> <p><b>Before</b></p> <ol style="list-style-type: none"> <li>Select an idea, strategy, and/or representation for focused instruction by: <ol style="list-style-type: none"> <li>choosing a string of related problems, or</li> <li>choosing a problem with a string of related questions, or</li> <li>choosing a string of related tasks</li> </ol> </li> </ol> <p><b>During</b></p> <ol style="list-style-type: none"> <li>Activate student's background knowledge from Develop Understanding phase.</li> <li>Launch and clarify the task(s).</li> </ol> <p><b>STUDENT ROLE:</b></p> <ol style="list-style-type: none"> <li>Actively listen.</li> <li>Ask clarifying questions.</li> <li>Reflect on experiences from previous phase</li> </ol>	<p><b>PURPOSE:</b></p> <p>Engage students in task(s) to solidify understanding and gain ownership of the selected idea, strategy, and/or representation.</p> <p><b>TEACHER ROLE:</b></p> <p><b>Before</b></p> <ol style="list-style-type: none"> <li>Anticipate student thinking and misconceptions and plan responses to guide focused discussion.</li> <li>Determine structure of teaching cycle(s) and student grouping (individuals, pairs, small groups, whole group).</li> </ol> <p><b>During</b></p> <ol style="list-style-type: none"> <li>Facilitate and direct student understanding and ownership by: <ol style="list-style-type: none"> <li>exposing and eliminating misconceptions, and</li> <li>asking questions to: <ul style="list-style-type: none"> <li>prompt,</li> <li>clarify,</li> <li>guide,</li> <li>scaffold,</li> <li>probe, and/or</li> <li>connect mathematical thinking.</li> </ul> </li> </ol> </li> <li>Continually refocus student thinking.</li> </ol> <p><b>STUDENT ROLE:</b></p> <ol style="list-style-type: none"> <li>Engage in task.</li> <li>Raise "how", "why", "what if", "so what" and "does this make sense" questions.</li> <li>Question, explain, and justify individual or group work using proper vocabulary.</li> <li>Make connections among the string of related problems, questions, or tasks.</li> <li>Make connections with previous learning.</li> </ol>	<p><b>PURPOSE:</b></p> <p>Use student understanding of ideas, strategies, and representations to move students to an emerging understanding of concepts, algorithms, and tools.</p> <p><b>TEACHER ROLE:</b></p> <p><b>Before</b></p> <ol style="list-style-type: none"> <li>Purposefully structure the focused discussion of ideas, strategies, and/or representations.</li> </ol> <p><b>During</b></p> <ol style="list-style-type: none"> <li>Ask probing/directed questions to draw out explicit and specific connections.</li> <li>Confirm correct thinking.</li> <li>Use direct instruction as appropriate.</li> <li>Use language, conventions, and symbols of mathematicians.</li> <li>Assess student understanding.</li> <li>Help students recognize emerging concepts, algorithms, and tools.</li> </ol> <p><b>After</b></p> <ol style="list-style-type: none"> <li>Determine the next phase of the learning cycle: <ul style="list-style-type: none"> <li>Solidify Understanding phase, or</li> <li>return to Develop Understanding phase with newly surfaced ideas, or</li> <li>move to Practice Understanding phase</li> </ul> </li> </ol> <p><b>STUDENT ROLE:</b></p> <ol style="list-style-type: none"> <li>Correctly use mathematical language, conventions, and symbols.</li> <li>Explain and justify knowledge.</li> <li>Describe connections between previous and current knowledge.</li> <li>Generalize knowledge along a continuum from specific to abstract.</li> <li>Transfer knowledge to new situations.</li> </ol>

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## PRACTICE UNDERSTANDING

The goal of Practice Understanding is to allow students to refine and acquire fluency with concepts, algorithms, and tools which leads to the development of generalizations, procedures, and models.

LAUNCH	EXPLORE	DISCUSS
<p><b>PURPOSE:</b> Pose a task that re-engages students with one or more concepts, algorithms, or tools to acquire fluency as defined by:</p> <ul style="list-style-type: none"> <li>• accuracy</li> <li>❖ efficiency</li> <li>❖ flexibility, and/or</li> <li>❖ automaticity</li> </ul> <p><i>Note: Practice leads to uniqueness as well as sameness.</i></p> <ul style="list-style-type: none"> <li>❖ The individual refinement of concepts, algorithms, or tools occurs simultaneously as fluency is acquired.</li> <li>• Students will develop common generalizations, procedures and models.</li> <li>•</li> </ul> <p><b>TEACHER ROLE:</b> <b>Before</b></p> <ol style="list-style-type: none"> <li>1. Identify the concept, algorithm, or tool to be practiced.</li> <li>2. Select or design a vehicle with appropriate constraints to drive the practice, i.e., routines, games, worksheets, reviews, 10 minute math, etc., or</li> <li>3. Embed the practice in the task of Develop Understanding or Solidify Understanding.</li> </ol> <p><b>During</b></p> <ol style="list-style-type: none"> <li>4. Connect the task to student's previous work.</li> <li>5. Launch and clarify the task(s).</li> </ol> <p><b>STUDENT ROLE:</b></p> <ol style="list-style-type: none"> <li>2. Actively listen.</li> <li>3. Ask clarifying questions.</li> <li>4. Reflect on experiences from previous phases.</li> </ol>	<p><b>PURPOSE:</b> Engage students in task(s) with appropriate constraints to hone, shape, and maintain concepts, algorithms, and tools.</p> <p><b>TEACHER ROLE:</b> <b>Before</b></p> <ol style="list-style-type: none"> <li>1. The teacher determines when to monitor student work for accuracy, efficiency, and flexibility, e.g. during or after the exploration.</li> </ol> <p><b>When Monitoring During the Exploration</b></p> <ol style="list-style-type: none"> <li>1. Monitor student work for fluency by: <ul style="list-style-type: none"> <li>• asking brief questions</li> <li>• eavesdropping on conversations, and</li> <li>• visually scanning student work.</li> </ul> </li> <li>2. Monitor student work for opportunities for refinement by: <ul style="list-style-type: none"> <li>• Asking questions to help students become more aware of what they are thinking and/or doing</li> <li>• Encouraging efficient or flexible use of strategies</li> </ul> </li> <li>3. Determine when to move to individual or group discussions</li> </ol> <p><b>When Monitoring After Exploration</b></p> <ol style="list-style-type: none"> <li>1. review student work for fluency by: <ul style="list-style-type: none"> <li>• Correcting student work for accuracy,</li> <li>• Looking for common themes (conceptions and misconceptions) across samples of students' work.</li> </ul> </li> </ol> <p><b>STUDENT ROLE:</b></p> <ol style="list-style-type: none"> <li>2. Reflect on work by asking: <ul style="list-style-type: none"> <li>• "Is this accurate?"</li> <li>• "Do I understand this?"</li> <li>• "Can I explain this?"</li> <li>• "Where would I use this?"</li> </ul> </li> </ol>	<p><b>PURPOSE:</b> Give students personalized feedback that leads to independent fluency and to move students to an emerging understanding of definitions, properties, procedures, and models.</p> <p><b>TEACHER ROLE:</b> <b>Before</b></p> <ol style="list-style-type: none"> <li>1. The teacher needs to be aware of the possible refinement that can occur during practice and ask questions to guide, mentor, and document this refinement.</li> </ol> <p><b>When Giving Feedback During Exploration</b></p> <ol style="list-style-type: none"> <li>1. Coach and mentor student work.</li> <li>2. Provide individualized feedback.</li> <li>3. Reinforce communication skills and computation.</li> <li>4. Help students recognize emerging generalizations, procedures, and models.</li> </ol> <p><b>When Giving Feedback After Exploration</b></p> <ol style="list-style-type: none"> <li>1. Provide individualized feedback.</li> <li>2. Identify emerging generalizations, procedures, and models.</li> </ol> <p><b>After</b></p> <ol style="list-style-type: none"> <li>1. Determine the next phase of the learning cycle <ul style="list-style-type: none"> <li>• remain in Practice Understanding,</li> <li>• return to Solidify Understanding , or</li> <li>• move to Develop Understanding.</li> </ul> </li> </ol> <p><b>STUDENT ROLE:</b></p> <ol style="list-style-type: none"> <li>1. Practice standard and invented algorithms, problem solving strategies, multiple representations, higher-level thinking, communication and fact recall.</li> <li>3. Increase efficiency, flexibility, automaticity and ability to justify work.</li> </ol>

