Name: **2009-05-11**
Algebra 2

Word Problems: Exponential vs. Linear

I. Decide whether each of the following situations is linear, exponential, or neither. Define a function that represents each situation.

1. Ali reads at a rate of 25 pages per hour. (Let x be the number of hours and $f(x)$ be the number of pages read.)
2. A bank account starting at $100 gains 3% every year. (Let x be the number of years and $f(x)$ be the value of the account.)
3. Avdeep runs 4 miles every hour. . (Let x be the number of hours and $f(x)$ be the number of miles.)
4. Peter tosses a ball into the air and its height is modeled by the function $h(t) = -16t^2 + 10t + 4$. (Ok, the function is already given here.)
5. A new car worth $20,000 loses 30% of its value every year. (Let x be the number of years and $f(x)$ be the value of the car.)
6. Every time a 10 cm x 10 cm piece of paper is folded, it becomes half as big. (Let x be the number of folds and $f(x)$ be the area of the folded paper.)
7. Connor starts with $75 and spends $15 every week. (Let x be the number of weeks and $f(x)$ be the amount he has left.)

II. Carefully solve each of the following problems. Pay attention linear versus exponential models.

1. In the year 1980, company A's sales were $5000 and company B's sales were $1400. Company A's sales grew by $400 each year. Company B's sales grew by 15% each year.
   a. Define a function $A(t)$ that shows company A's sales t years after 1980.
   b. Define a function $B(t)$ that shows company B's sales t years after 1980.
   c. Whose sales were higher in 1993, and by how much?
   d. Graph both functions on your calculator. When did company B's sales first surpass company A's sales?
   e. Write an equation that you could solve to determine when company A's sales were $10,000. Write an equation that can be solved to determine when company B's sales were $10,000. Solve the first one by hand and use your calculator to solve the 2nd one.

2. A new luxury automobile costs $50,000. Assume its value after seven years is $22,000.
   a. Assume the function describing the car's value when it is x years old is linear. Write an equation for it (in slope-intercept form). Hint: the 2 points you know are (0,something) and (7, something).
   b. What is the slope and what does it mean? What is the intercept and what does it mean?
   c. Assume instead that the value fell by a constant percentage rate each year. What was the percentage decline in value?
   d. Write an exponential function describing the car's value when it is x years old.
   e. Use each model to calculate the car's value when it is 3 years old and when it is 13 years old.
   f. Which model do you think is better and why?
   g. Use each model to determine when the car's value first hits $10,000. You will need your calculator for the exponential model.
   h. Use each model to determine when the car is worthless.

3. The population of a small country is currently 2 million and is growing at the rate of 5% per year. The country currently produces enough food to feed 4 million people. Because of increases in

population and farming techniques, each year the food supply increases by enough to feed 0.3 million more people per year.

a. Write a function *p(t)* showing the population (in millions) *t* years from now.
b. Write a function *f(t)* showing the food supply (for millions of people) *t* years from now.
c. Twenty years from now, what will the population be and what will the food supply be?
d. Graph both functions on your calculator. When will the population surpass the food supply?
e. What is the meaning of $f(t) = 6$?

4. The value of an object was $4000 in 1980. In 2007, it was worth $7800. The annual percent growth has been constant.

a. What is the annual percent growth?
b. Write an equation showing the object's value t years after 1980.
c. When will its value first hit $12,000?
d. What was its value in 1993?
e. What if this situation was linear, rather than exponential? In other words, suppose the value of the object increased by the same amount each year. Write a new equation giving the object's value t years after 1980. When will its value first hit $12,000? What was its value in 1993?

ANSWERS:
Section I:
1. Linear 2. Exponential 3. Linear 4. Neither (it's quadratic) 5. Exponential 6. Exponential 7. Linear.
Section II:
1a. $A(t) = 5000 + 400t$ b. $B(t) = 1400 \cdot 1.15^t$ c. $A(13) = 10{,}200$ $B(13) = 8614$; so A's by $1586. d. year 15, so 1995; (meet at t=14.66) e. $5000 + 400t = 10000$ $1400 \cdot 1.15^t = 10000$ company A: t=12.5 (mid 1992) company B: t=14.07 years
2a. $y = -4000x + 50000$ b. starts at $50,000 and falls by $4000 per year c. -11.1% per year d. $y = 50000 \cdot 0.889^x$ e. 3 years: linear=38,000 exp=35,130 13 years: linear=-2000 exp=$10,832. f. I like exponential: value never becomes negative g. linear 10 years; exp is 13.68 years h. linear: 12.5 years; exponential: never
3a. $p(t) = 2 \cdot 1.05^t$ b. $f(t) = 4 + 0.3t$ c. $p(20) = 5.31$ mill $f(20) = 10$ mill d. in 44.16 years
4a. $7800 = 4000(1 + r)^{27}$ so r=2.5% b. $V(t) = 4000(1.025)^t$ c. 44.49 years
d. $4000(1.025)^{13} = 5514$ e. $V(t) = \frac{3800}{27}t + 4000$, in 2037, $5829